

Teacher's User Guide for:
Number Line Touch: Multiplication

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Overview and Purpose

Number Line Touch: Multiplication is a mobile mathematics learning app designed for the iPad. At each level, users are presented a number line based riddle to find missing numbers. To facilitate this, they can drag and drop different rods onto the number line to help them measure and count the intervals.

The app is designed for elementary aged students to scaffold learning of multiplication and division, as well as develop a meaningful understanding of the number line. The number line itself is a symbolic and visual representation. By allowing users to drag and drop different rods onto the number line in the app, users are given a concrete/tactile experience for making sense of number lines.

The levels in *Number Line Touch: Multiplication* are designed based on educational research on how children develop **meaningful** understandings of multiplication and division. Level 1 scaffolds learning how to skip-count. Initially, children may count by 1s, and Level 1 includes five sub-levels to give users experiences to learn to transition from counting by 1s to skip-counting (e.g., 3, 6, 9, ...). Replaying each level will randomly generate a new number line riddle, allowing for users to practice with a wide range of counting strategies.

Level 2 focuses on improving users' flexibility with skip-counting in order to prepare users for formal multiplication and division tasks. As with Level 1, replaying each level randomly generates a new number line riddle.

At Level 3, users must build off of their experiences skip-counting to use formal multiplication and division to solve number line riddles. Level 4 includes riddles with increased difficulty, while Level 5 introduces formal algorithms for multiplication and division (i.e., long division and multi-digit multiplication).

The current version of Number Line Touch: Multiplication only includes Level 1. However, future levels are in development and will be released through updates of the app.

This user guide includes information about the app, alignment between levels and content standards, and extension activities for the classroom at each level. As the app is updated, additional material and description will be added to this user guide.



Alignment with Common Core State Standards for Mathematics (CCSS-M)

Each level includes sub-levels that, collectively, scaffold a particular skill for developing multiplicative reasoning. The collective sub-levels meet specific content standards, which are provided below for convenience.

Level 1

Counting on the Number Line

Grade 2 2.NBT.B.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.

2.MD.B.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Level 2

Skip-Counting on the Number Line

Grade 3 3.OA.A.1

Interpret products of whole numbers.

3.OA.B.5

Apply properties of operations as strategies to multiply and divide.

3.OA.D.9

Identify arithmetic patterns, and explain them using properties of operations.

Grade 4 4.OA.C.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Level 3

Multiplying on the Number Line

Grade 3 3.OA.A.1

Interpret products of whole numbers.

3.OA.A.2

Interpret whole-number quotients of whole numbers.

3.OA.B.5

Apply properties of operations as strategies to multiply and divide.

3.OA.B.6

Understand division as an unknown-factor problem.

3.OA.C.7

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, or properties of operations.

3.OA.D.9

Identify arithmetic patterns, and explain them using properties of operations.

Grade 4 4.OA.A.1

Interpret a multiplication equation as a comparison.

4.OA.C.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Additional Standards Alignment with Later Levels is Forthcoming.

Level 4: More Multiplying on the Number Line

Level 5: Algorithms on the Number Line



How to Use the App

Each level includes an overarching targeted skill with sublevels to help scaffold that skill. For example, Level 1 targets children’s learning how to skip count by providing experiences with the number line that moves from counting by 1s to counting onto or down from, counting by groups of 1s, and then an introduction to skip-counting in Level 1.5. There are two features of *Number Line Touch: Multiplication* that are key for teachers and/or parents to scaffolding their child’s play with the app: 1) dragging the rods to count directly on the number line, and 2) replaying the levels to gain experience with different numbers.

1. Dragging rods onto the number line.

Encourage children to drag the rods onto the number line to count and find the missing numbers. This helps children begin to make sense of the number line. Quite often, children confuse the intervals (spaces between the markings on the number lines) and the markings themselves when counting. By dragging the rods onto the number line, children begin make sense, intuitively, of what the intervals mean. The reason this skill is often so difficult for children to initially grasp is that the number line is both a pictorial and symbolic representation. Put simply, children often have no point of reference for making sense of the number line. Using the rods with the number line is, therefore, a critically important feature of the app to help children develop mathematically.

2. Replaying the levels.

A key feature of *Number Line Touch: Multiplication* is that each time a level is replayed, the number riddle changes. So, if a child is playing Level 1.3, they may be asked to begin counting by 3’s (dragging 1-unit rods onto the number line) to find different numbers. If they replay they level, they will be asked to find different missing numbers and the interval will change: so they may be asked to begin counting by 5’s instead. Children can restart a level while they are playing it. However, if they exit, then re-enter a level, the number riddle changes via a random number generator.

It is important to make sure that children try and replay certain levels so they gain more experience with different numbers. By ensuring that the number riddle is never the same when replaying back-to-back, the app design encourages children to attend to precision in how they use the number line, and in how they think multiplicatively.

A Final Note about Using the App

The app itself is a starting point for engaging children to think multiplicatively. The different scaffolded levels are useful and important, but it is meant to be one of many kinds of



experiences children should engage in to learn multiplication and division. Within this user guide are some example activities designed for classroom teachers to use in conjunction with, but externally from, the app. However, many other useful and thoughtful activities can, and should, be used with children as they develop their multiplicative reasoning.



Level 1 Tasks

There are seven tasks included as extensions for Level 1 of the *Number Line Touch: Multiplication* app. The task name corresponds to the sub-level in the app. So, Task 1.1 can be used as an in-class extension for Level 1.1 in the app, Task 1.2 can be used with Level 1.2, and so on. Therefore, each task should also correspond with the content standards identified on pp. 3-4. Each task includes a paper-based number line designed to be used with concrete manipulatives. To use a task, you will need to do the following:

- Print the task you wish to use. It is important that in the print settings you select ACTUAL SIZE. If another setting is selected, the number lines may not function with the manipulatives as designed.
- Obtain a set of either the *ones-cubes for base ten blocks* **or** *Cuisenaire rods*. Each manipulative will work in a similar way as the rods in the *Number Line Touch* app.

As you engage children with each task, encourage them to use the manipulative on the paper-based number line to find the missing numbers:

- If using ones-cubes from base ten blocks, children may need to consider a group of ones rods as representing something unique (e.g., three ones-rods may fill the space of a five-length interval requiring children to count by 5's with three ones-rods stacked together).
- If using Cuisenaire rods, encourage children to use the rods in the way that makes the most sense to them.

As children find the missing values, ask them to write their numbers in the chart below the number line, and then to write an equation or expression (number sentence) that represents how they found that number. These charts can be used in class for discussion about patterns children observe, and to compare strategies.

A few additional notes:

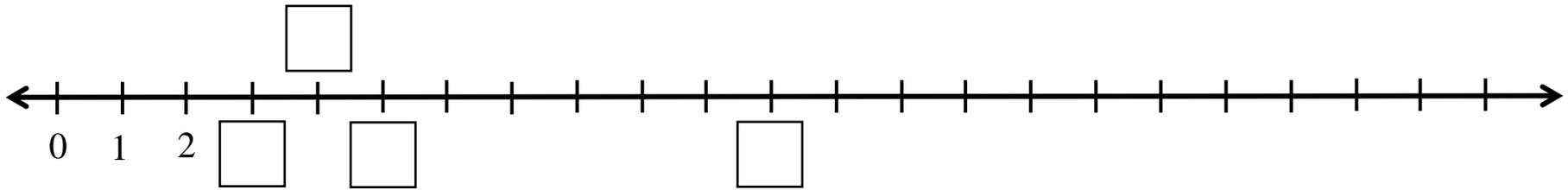
Level 1.5 tasks can be identical to many Level 2 tasks. The difference is how instruction is scaffolded. Specifically, students may not be able to skip-count with ease. Thus, Level 1.5 tasks are designed to allow for them to engage in productive struggle around the content. Teachers using these tasks to correspond with Level 1.5 in the app should expect some miscounting when skip-counting is attempted (e.g., 3, 6, 7, 8, 9...). These tasks should allow for a safe way of engaging in this content. One way to help scaffold this is to allow students to construct their own iterable unit rods out of paper. For example, on Task 1.5 A, students can create a 1 cm rod on construction paper. On one side, a teacher can have students partition (by drawing a line) so that it represents two units, while the other side remains blank. Having the students construct these on their own encourages them to consider the unit as two 1s, but also as one 2.

For teachers who wish to have additional number line activities, a sheet of blank number lines is provided on page 16 to adapt as needed.



Task 1.0

Find the missing numbers below.



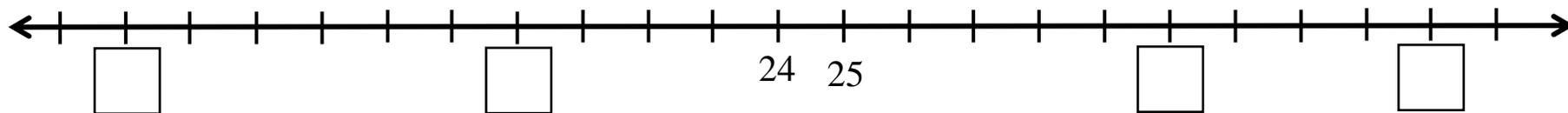
For each number you find, write a math sentence that describes how to go from 0 to that new number:

Missing number:	Math Sentence:



Task 1.1

Find the missing numbers below.



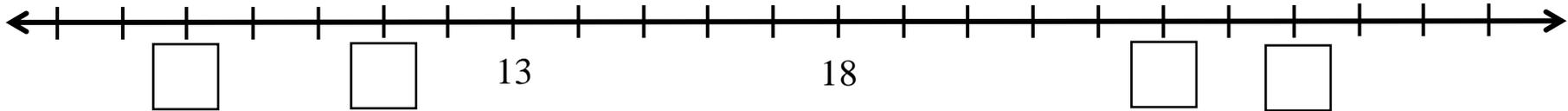
For each number you find, write a math sentence that describes how to go from 24 to that new number:

Missing number:	Math Sentence:



Task 1.2

Find the missing numbers below.



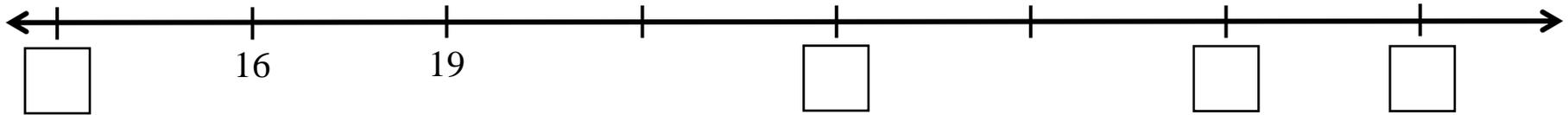
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 1.3

Find the missing numbers below.



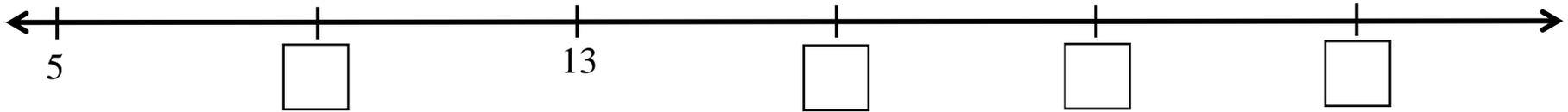
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 1.4

Find the missing numbers below.



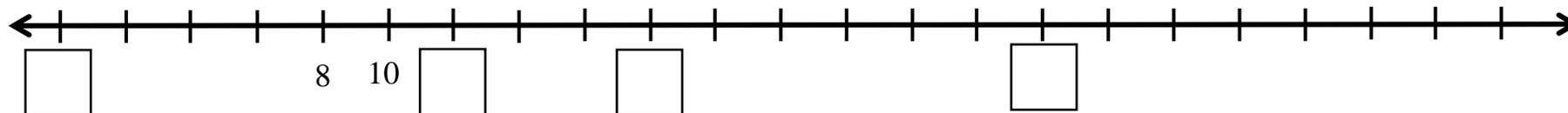
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 1.5 - A

Find the missing numbers below.



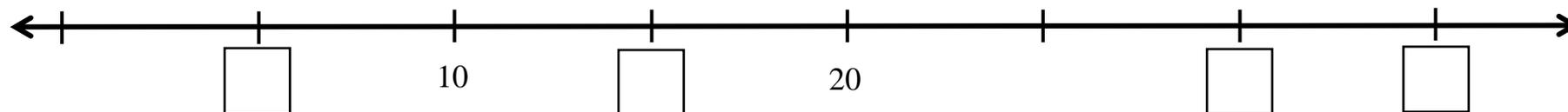
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 1.5 - B

Find the missing numbers below.



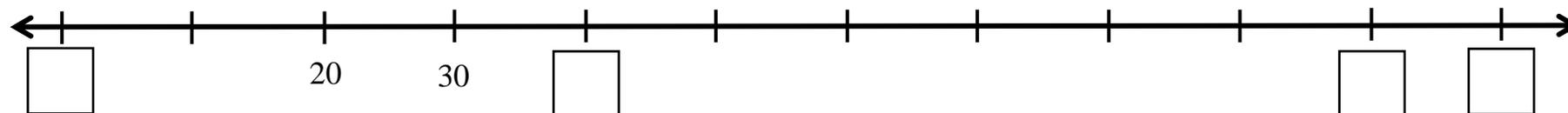
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 1.5 - C

Find the missing numbers below.



For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:

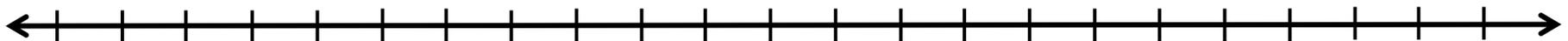
Blank Number Lines for Teacher-Created Extensions



Blank number line



1 cube spaces



2 cube spaces



3 cube spaces



4 cube spaces



Level 2 Tasks

There are three tasks **currently** included as extensions for Level 2 of the *Number Line Touch: Multiplication* app. Additional tasks are forthcoming as the app is further developed. The task name corresponds to the sub-level in the app. So, Task 2.1 can be used as an in-class extension for Level 2.1 in the app, and so on. Therefore, each task should also correspond with the content standards identified on pp. 3-4. Each task includes a paper-based number line designed to be used with concrete manipulatives. To use a task, you will need to do the following:

- Print the task you wish to use. It is important that in the print settings you select ACTUAL SIZE. If another setting is selected, the number lines may not function with the manipulatives as designed.
- Obtain a set of either the *ones-cubes for base ten blocks* **or** *Cuisenaire rods*. Each manipulative will work in a similar way as the rods in the *Number Line Touch* app.

As you engage children with each task, encourage them to use the manipulative on the paper-based number line to find the missing numbers:

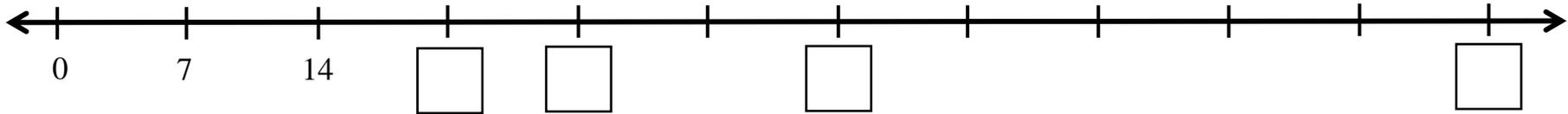
- If using ones-cubes from base ten blocks, children may need to consider a group of ones rods as representing something unique (e.g., three ones-rods may fill the space of a five-length interval requiring children to count by 5's with three ones-rods stacked together).
- If using Cuisenaire rods, encourage children to use a single rod that matches the interval length. This encourages children to become more fluent in their skip-counting.

As children find the missing values, ask them to write their numbers in the chart below the number line, and then to write an equation or expression (number sentence) that represents how they found that number. These charts can be used in class for discussion about patterns children observe, and to compare strategies.

Task 2.6 includes three number lines with the goal of finding how long each interval is worth. This task corresponds with division. Thus, when working with children on this task, teachers are encouraged to have students write a multiplication equation and division equation that corresponds to the number line. This will help students begin to consider the inverse relationship between multiplication and division more formally.

Task 2.0

Find the missing numbers below.



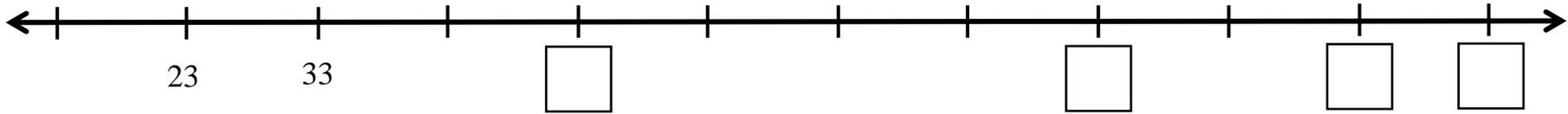
For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:



Task 2.1

Find the missing numbers below.



For each number you find, write a math sentence that describes how you found the new number:

Missing number:	Math Sentence:

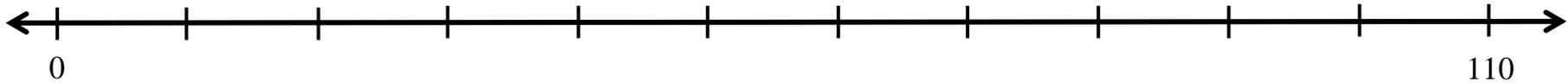


Task 2.6

How long is each space on the number line below? How do you know?



How long is each space on the number line below? How do you know?



How long is each space on the number line below? How do you know?

